**ECE 581 Homework 7**

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1. Problem 5-1

Given that we have a known signal image and an unknown image to be determined. According to the algorithm in Hmwk2, we could transform two matrixes in to a single variable .

is a single element of , is a single element of

is our observation variable and according to the result in Hmwk2,

So, the likelihood ratio supposed to be , the Ln likelihood ratio supposed to be:

.

According to the transformation of , we could get the pdf of :

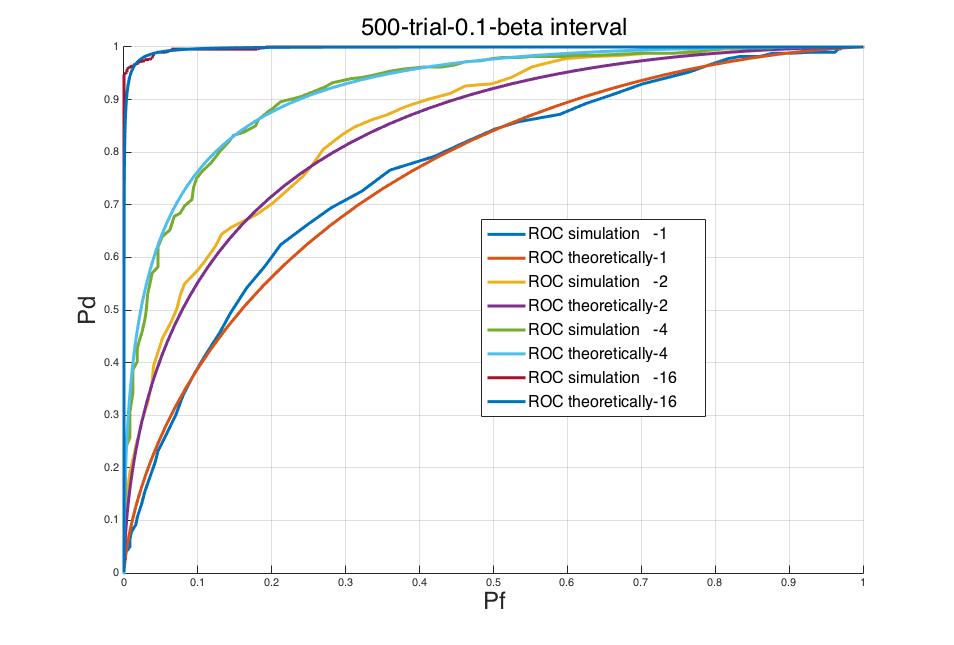
According to the equations above, we could calculate the theoretical ROC.

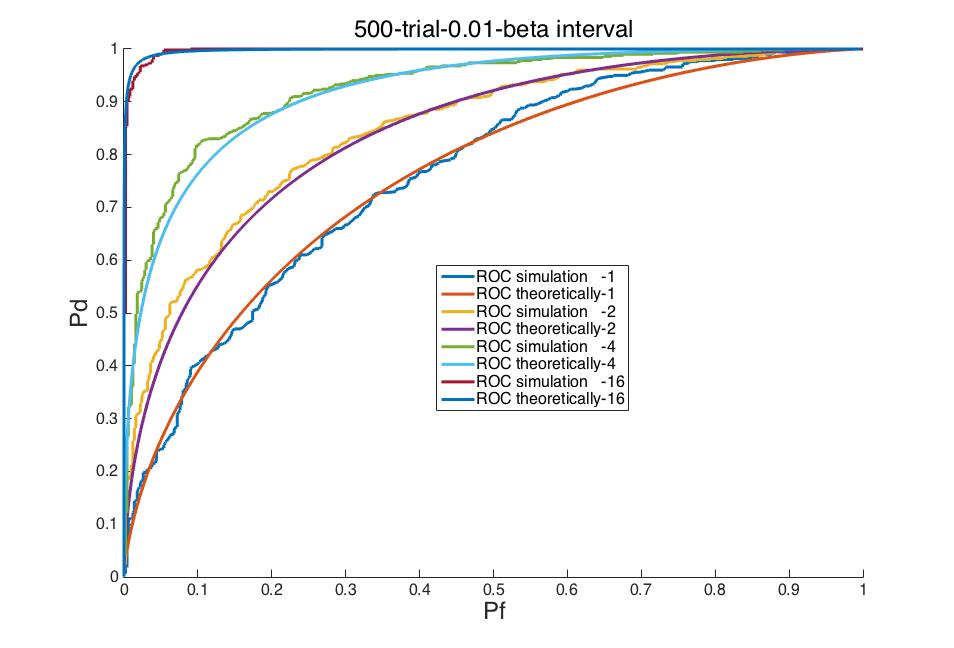
2. Problem 5-2

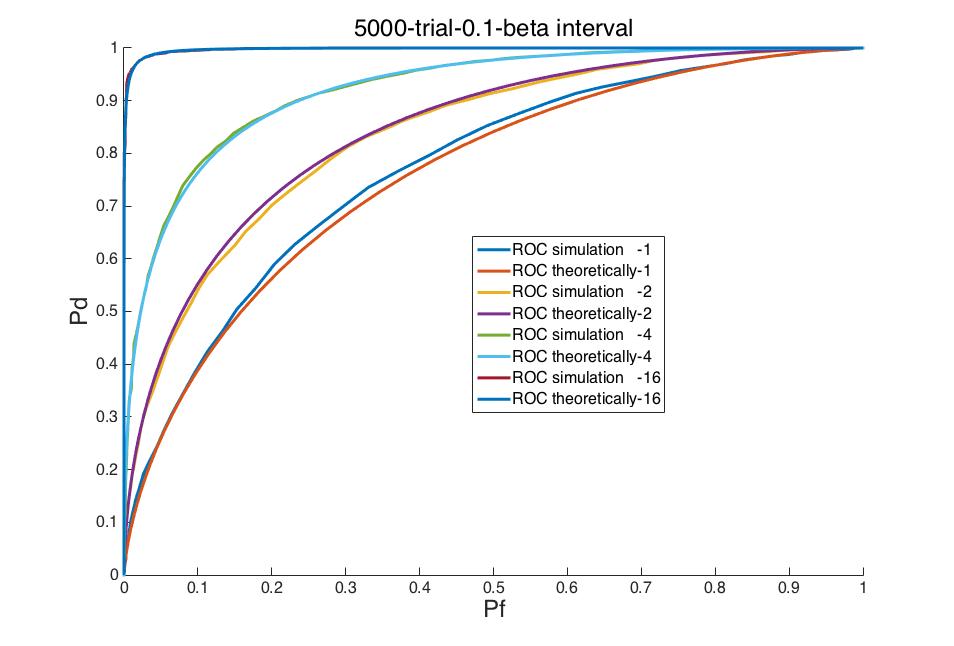
The brute-force computer simulation in this problem is to generate 500 test images with only noise () and 500 test images with noise and signal () and calculate the observation value of each image. Because the noise and signal is statistically independent. More than that, noise is according to N(0,1) distribution. So, the value , is also according to the pdf above. 500 trials could sketchily describe the distribution of value under each hypothesis with the number of values of variable z in each interval. With these values of z, we could get numbers of values which are also according to the pdf above.

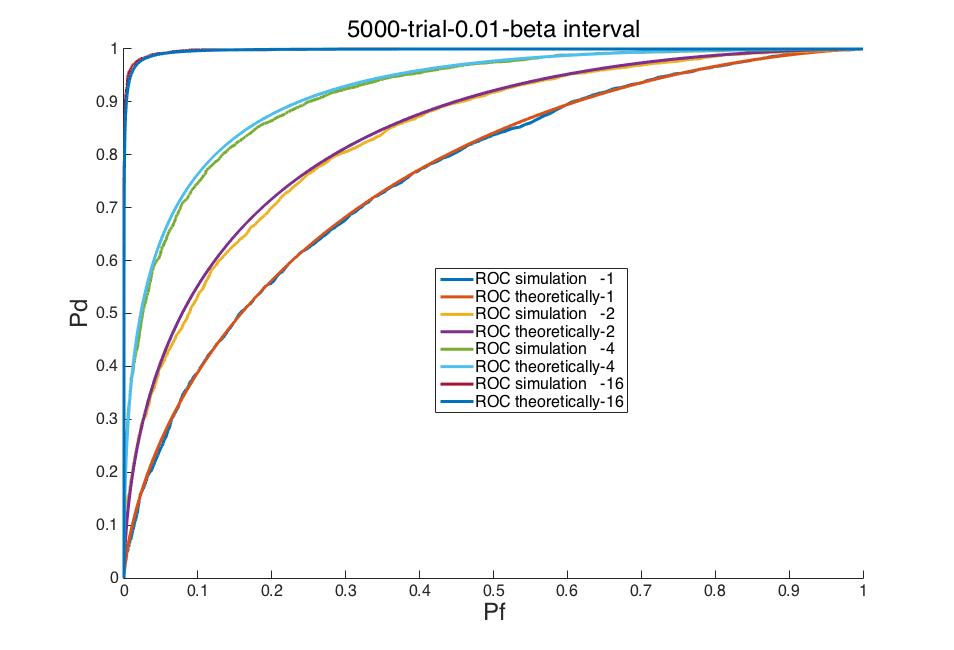
So, for brute-force computer simulation, =the number of values of , is threshold. =the number of values of , is threshold. At last, connect different pairs (,) with different , which is so called ROC curve.

For theoretical result, ROC curve is also generated by different pairs of (,)

 ,







As we could see in four pictures above, the number before ‘trial’ means the number of trials, the number before ‘beta interval’ means the interval of different .

* **Why is ROC closer to the upper-left corner with higher**

**Of course we could understand is a kind of way to describe SNR.** In the picture, we could see the ROC curve under different SNR of input images. **The higher the SNR is, the closer to the upper-left corner the ROC curve is.**

As we all know SNR is a way to describe the sensitivity of two different hypothesizes. The higher the SNR is, the higher the sensitivity is, the higher theis for each specific .

* **Why isn’t ROC simulation curve equal to ROC theoretically**

Because the higher the number of trials is, the closer to the odds the frequency of in some region is. When the number of trials is infinite, the frequency is equal to odds. That is also why the ROC simulation curve is closer to ROC theoretically comparing with 500 trials and 5000 trials.

There is another factor to affect the error between ROC simulation and ROC theoretically which is the interval of when we drew the ROC curve. The interval of is resolution of our graph. The smaller the is, the higher the resolution is. That is why the ROC simulation curve is closer to ROC theoretically comparing with 0.1 beta interval and 0.01 beta interval.

* **Why isn’t ROC simulation curve smooth**

Because when the number of trials is not big enough, the distribution of is not very uniformly. That is why a small increase would change or a lot. Comparing with 500 trials and 5000 trials, the latter graph is much more smooth. On the other hand, the distribution of is more uniformly with a larger interval. That is why the curve with 0.1 interval is more smooth than 0.01 interval.

Hmwk7\_force\_simulation.m

%================================================

% ECE 581 Hmwk7

% Shengxin Qian

%================================================

clear

set(figure,'NumberTitle','off','Name','abc');

for Es=[1,2,4,16] %keep variance of noise=1 and set Es=1,2,4,16

s=signal\_image\_generator(1024,Es); %generate signal image according to required Es

sum\_varianceH0=sum\_variance\_H0(s,1024,1); %calculate the required variance(H0) for algorithm in Hmwk2

sum\_varianceH1=sum\_variance\_H1(s,1024,1); %calculate the required variance(H1) for algorithm in Hmwk2

sum\_meanH1=sum\_mean\_H1(s,1024,1); %calculate the required mean(H1) for algorithm in Hmwk2

trial\_number=5000; %the experiment time

z\_H0=normrnd(0,sqrt(sum\_varianceH0),1,trial\_number);%generate the output(H0) of algorithm in Hmwk2

z\_H1=normrnd(sum\_meanH1,sqrt(sum\_varianceH1),1,trial\_number);%generate the output(H1) of algorithm in Hmwk2

lnlambda\_H0=z\_H0-0.5\*sum\_meanH1; %the lnlambda\_H0 output according to the problem 5-1

lnlambda\_H1=z\_H1-0.5\*sum\_meanH1; %the lnlambda\_H1 output according to the problem 5-1

index=1;

for beta=min(lnlambda\_H0):0.01:max(lnlambda\_H1)

Pf(index)=sum(lnlambda\_H0>=beta); %Pf in experiment

Pd(index)=sum(lnlambda\_H1>=beta); %Pd in experiment

Pft(index)=1-normcdf(beta,-0.5\*sum\_meanH1,sqrt(sum\_varianceH0));%Pf theoretically

Pdt(index)=1-normcdf(beta,0.5\*sum\_meanH1,sqrt(sum\_varianceH1)); %Pd theoretically

index=index+1;

end

hold on

plot(Pf/trial\_number,Pd/trial\_number);

plot(Pft,Pdt);

end

signal\_image\_generator.m

function y=signal\_image\_generator(n,Es)

s=sqrt(Es/n);

y=eye(n)\*s;

sum\_mean\_H1.m

function y=sum\_mean\_H1(s,n,single\_variance)

y=0;

for i=1:n

for j=1:n

y=y+s(i,j)^2;

end

end

y=y/single\_variance;

sum\_variance\_H0.m

function y=sum\_variance\_H0(s,n,single\_variance)

y=0;

for i=1:n

for j=1:n

y=y+s(i,j)^2;

end

end

y=y/single\_variance;

sum\_variance\_H1.m

function y=sum\_variance\_H1(s,n,single\_variance)

y=0;

for i=1:n

for j=1:n

y=y+s(i,j)^2;

end

end

y=y/single\_variance;